

Noncommutativity from the string perspective: modification of gravity at a mm without mm sized extra dimensions

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ABSTRACT: We explore how the IR pathologies of noncommutative field theory are resolved when the theory is realized as open strings in background B -fields: essentially, since the IR singularities are induced by UV/IR mixing, string theory brings them under control in much the same way as it does the UV singularities. We show that at intermediate scales (where the Seiberg-Witten limit is a good approximation) the theory reproduces the noncommutative field theory with all the (un)usual features such as UV/IR mixing, but that outside this regime, in the deep infra-red, the theory flows continuously to the commutative theory and normal Wilsonian behaviour is restored. The resulting low energy physics resembles normal commutative physics, but with additional suppressed Lorentz violating operators. We also show that the phenomenon of UV/IR mixing occurs for the graviton as well, with the result that, in configurations where Planck's constant receives a significant one-loop correction (for example brane-induced gravity), the distance scale below which gravity becomes non-Newtonian can be much greater than any compact dimensions.

KEYWORDS: Non-Commutative Geometry, Models of Quantum Gravity, D-branes.

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1. Introduction

Gauge theories in which the coordinates are noncommuting,

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \tag{1.1}$$

are interesting candidates for particle physics, with curious properties (for general reviews of noncommutative gauge theories see refs. [1–3]). One whose consequences we would like to understand a little better is ultra-violet(UV)/infra-red(IR) mixing [4, 5]. This is a phenomenon which gives rise to various pathologies in the field theory, making it, at best, difficult to understand. In this paper we set about examining UV/IR mixing from the point of view of string theory with a background antisymmetric tensor ($B^{\mu\nu}$) field, which provides a convenient UV (and hence IR) completion. Along the way, as well as seeing how the pathological behaviour is smoothed out, we will outline the characteristic phenomenology of this general class of theories in the deep IR (i.e. at energy scales lower than those where noncommutative field theory is a good description): they resemble the $B = 0$ theories but with Lorentz violating operators which can be taken parametrically and continuously to zero by reducing the VEV of $B^{\mu\nu}$. As a bi-product we also show that the UV/IR mixing phenomenon extends to the gravitational sector (although a field theoretical interpretation for UV/IR mixing in gravity is difficult to obtain). This allows

the curious possibility that gravity may be non-Newtonian on much longer length scales than those associated with the compact dimensions.

Because UV/IR mixing, and the particular problems and phenomena to which it gives rise, are rather subtle, we begin now with a detailed discussion of exactly what questions we would like the string theory to answer, after which we restate our findings in more precise terms. UV/IR mixing has its origin in the fact that the commutation relations intertwine large and small scales. At the simplest level, in a gedanken experiment where x_1 and x_2 do not commute, the uncertainty relation $\Delta x_1 \Delta x_2 \sim i\theta^{12}$ together with the usual Heisenberg uncertainty $\Delta x_1 \Delta p_1 \sim i$ imply $\Delta x_2 \sim -\theta^{12} \Delta p_1$: short distances in the 1 direction are connected to small momenta in the 2 direction and vice versa. At the field theory level, this intertwining of UV and IR leads to the infamous phenomenon of UV/IR mixing in the non-planar Feynman diagrams: nonplanar diagrams are regulated in the UV but diverge in the IR. Essentially, contrary to the standard picture of the Wilsonian effective action, heavy modes do not decouple in the IR so that, for example, trace U(1) factors of the gauge group run to a free field theory in the IR even if there are no massless excitations [6–10].

The agent responsible for these unusual and challenging features of noncommutative gauge field theories is the Moyal star product,

$$(\phi * \varphi)(x) \equiv \phi(x) e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu\overrightarrow{\partial}_\nu} \varphi(x), \tag{1.2}$$

used in their definition. It induces a phase factor $\exp \frac{i}{2}k \cdot \theta \cdot q$ in the vertices, where k is an external momentum and q is a loop-momentum. This oscillating phase regulates the nonplanar diagrams in the UV, which can most easily be expressed using Schwinger integrals: for example the one-loop contribution to vacuum polarization takes the form (c.f. [6–9, 11, 12])

$$\Pi_{\mu\nu}(k) \sim \int \frac{dt}{t} e^{-\frac{\tilde{k}^2}{4t}} \dots \tag{1.3}$$

where $\tilde{k}^\mu = \theta^{\mu\nu} k_\nu$ and the ellipsis stands for factors independent of \tilde{k} . The exponential factor in the integrand is a regulator at $t \sim \tilde{k}^2 \sim k^2/M_{NC}^4$, where we define the generic noncommutativity scale by $\theta^{\mu\nu} = \mathcal{O}(M_{NC}^{-2})$. Thus the diagram, which without this factor would be UV divergent, is regulated but only so long as $\tilde{k} \neq 0$. The result is that the UV divergences of the planar diagrams reappear as IR poles in \tilde{k} in the nonplanar diagrams.

These divergences are problematic. First they signal a discontinuity because the $\tilde{k} \rightarrow 0$ limit of the integrals is not uniformly convergent: physics in the limit $\theta \rightarrow 0$ does not tend continuously to the commutative theory. Moreover they lead to alarming violations of Lorentz invariance. For example, the lightcone is generally modified to a lightwedge [11, 13]. This is in sharp disagreement with observation. Furthermore in noncommutative gauge theory, the trace U(1) photon has a polarization tensor given by [5]

$$\Pi_{\mu\nu} = \Pi_1(k^2, \tilde{k}^2) (k^2 g_{\mu\nu} - k_\mu k_\nu) + \Pi_2(k^2, \tilde{k}^2) \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^4}, \tag{1.4}$$

where the additional term $\sim \Pi_2$ is multiplied by a Lorentz violating tensor structure. It is absent in supersymmetric theories [5], but since supersymmetry is broken, we expect it to be

at least of order M_{SUSY}^2 times by some factor logarithmic in \tilde{k} (where M_{SUSY} is a measure of the supersymmetry breaking). The result is a mass of order M_{SUSY} for certain polarizations of the trace-U(1) photon while other polarizations remain massless [14]. Gymnastics are then required to prevent this trace U(1) photon mixing with the physical photon.

Clearly then, the outlook from the perspective of field theory is gloomy; because the IR singularities are a reflection of the fact that field theory is UV divergent, any attempt to resolve them without modifying the UV behaviour of the field theory is doomed. With this understanding, the general expectation is for a more encouraging picture in a theory with a UV completion, such as string theory. A more precise argument is the following. First it is easy to appreciate that, without an explicit UV completion, noncommutative field theory is unable to describe physics in the IR limit. As noted in ref. [15] and in the specific context of string theory in ref [16, 17], UV/IR mixing imposes a IR cut-off given by $|k| > \Lambda_{IR} = \frac{M_{NC}^2}{\Lambda_{UV}}$. Inside the range

$$\Lambda_{IR}^{ij} \sim \frac{1}{|\theta_{ij}|\Lambda_{UV}} < |k| < \Lambda_{UV}, \tag{1.5}$$

the field theory behaves in a Wilsonian manner, in the sense that modes with masses greater than the UV cut-off do not (upto small corrections) affect the physics there. However outside this range the Wilsonian approach breaks down because modes above Λ_{UV} affect the physics below Λ_{IR} . Indeed this inequality makes it impossible to make statements about either the $\theta^{ij} \rightarrow 0$ limit or the $\tilde{k} \rightarrow 0$ limit within field theory. *In other words, a UV completion is needed not only to describe physics above Λ_{UV} but also physics below Λ_{IR} , and in particular to discuss the existence or otherwise of discontinuities there.* The picture is most obvious in the context of running of gauge couplings. Between Λ_{IR} and Λ_{UV} the effective action accurately describes the running of the trace U(1) gauge coupling regardless of what happens above Λ_{UV} . Below Λ_{IR} , UV physics intervenes. For example a period of power law running due to KK thresholds in the UV is mirrored by the "inverse" power law running in the IR. Now, the precise UV completion may take various forms, but suppose for example that it acts like a simple exponential cut-off, $e^{-\frac{\Lambda_{UV}}{4t}}$, in the Schwinger integral. The planar diagrams are regulated in the usual manner, but the nett effect of the noncommutativity for the nonplanar diagrams is that the UV cut-off Λ_{UV}^2 is replaced by $\Lambda_{\text{eff}}^2 = 1/(\tilde{k}^2 + \Lambda_{UV}^{-2})$ [4]. In this case when $\tilde{k} \ll \Lambda_{UV}^{-1}$ (i.e. when we are below the IR cut-off) we would have

$$\Pi_{\mu\nu} \approx \Pi_1(k^2, \Lambda_{UV}^{-2}) (k^2 g_{\mu\nu} - k_\mu k_\nu) + \Pi_2(k^2, \Lambda_{UV}^{-2}) \Lambda_{UV}^4 \tilde{k}_\mu \tilde{k}_\nu, \tag{1.6}$$

and normal Wilsonian behaviour would be restored, with the couplings matching those at the UV cut-off scale. Of course there is no reason to suppose that such a cut-off in any way resembles what actually happens in string theory, and to discuss the nature of the theory below Λ_{IR} requires full knowledge of the real UV completion. *What then are our general expectations for physics below Λ_{IR} ? Does it correspond to an effective field theory? If so, what happens to the Lorentz violating divergences in the IR?*

These are the precise questions we would like to explore, using a framework in which the noncommutative gauge theory is realized as a low-energy effective theory on D-branes [18 –

20, 1]. Our arguments are based on the two point function as calculated on D-branes in the background of a non-zero B -field [21, 22, 16, 17]. In such a theory, taking the zero slope limit in a particular way [1] ($\alpha' \rightarrow 0$ with $g_{\mu\nu} \sim \alpha'^2$) yields a noncommutative field theory in which the role of the noncommutativity parameter is played by the gauge invariant Born-Infeld field strength: indeed in this limit the open string metric and the noncommutativity parameter are given by [1]

$$G^{\mu\nu} = \left(\frac{1}{g-F} g \frac{1}{g+F} \right)^{\mu\nu} \quad (1.7)$$

with $F_{\mu\nu} = 2\pi\alpha' B_{\mu\nu}$, $B_{\mu\nu}$ being the (magnetic) field strength, and

$$\theta^{\mu\nu} = -2\pi\alpha' \left(\frac{1}{g-F} F \frac{1}{g+F} \right)^{\mu\nu} \quad (1.8)$$

respectively (we will henceforth restrict ourselves to noncommutativity in the space directions which we will label ij). The theory at finite α' provides a convenient UV completion of the noncommutative gauge theory. The UV "cut-off" acquires a physical meaning: it is the scale above which the noncommutative field theory description is invalid and string modes become accessible, and is of order

$$\Lambda_{UV} = 1/\sqrt{\alpha'}. \quad (1.9)$$

The IR "cut-off" is accordingly given by

$$\Lambda_{IR} = \sqrt{\alpha'} M_{NC}^2, \quad (1.10)$$

and, likewise, physics below this scale is best understood by performing a string calculation. We will rather loosely continue referring to the scale Λ_{IR} as the IR cut-off although of course we are chiefly interested in exploring the effective theory below it.

What we will show in this paper is that *the one-loop effective theory in the $k \rightarrow 0$ limit (including any threshold contributions) is the same as the commutative $\theta = 0$ theory*, and in particular there are no IR divergences. Below Λ_{IR} , physics differs from the $\theta = 0$ physics only by nonsingular residual effects that are calculable in any specific model, and we will estimate their magnitude. In addition we point out that the two point function of the graviton also gets stringy contributions at one-loop which can modify gravity right down to Λ_{IR} : if for example $M_{NC} \sim 1\text{TeV}$ and $M_s \sim M_{Pl}$, then gravity is modified at a mm even when there are no large extra dimensions. This is an effect equivalent to the one described for the gauge theory however there is no simple effective field theory description and it is difficult to understand in terms of "planar" and "nonplanar".

The rest of the paper is organized as follows. In the next section, we will discuss and determine the general form of UV/IR mixing in noncommutative field theory which is embedded in string theory. In section 3 and 4, the mentioned general characteristics of UV/IR mixing will be justified with explicit amplitude calculations based on bosonic and superstring models. In section 5, we will analyse how the graviton two point function is modified by noncommutativity. In section 6, we will discuss how noncommutativity in string theory may lead to a modification in the IR property of gravity. We will also discuss its phenomenological implications.

2. General remarks on UV/IR mixing in string theory

Assuming that the string theory amplitudes are finite (as issue to which we return in due course), it is natural that the IR singularities should be cured in much the same way as UV singularities are, since they are intimately related: they are essentially the same singularities. It is also natural that string theory should cure discontinuities afflicting the field theory; we certainly expect a string amplitude calculated at non-zero F , which is after all a rather mild background, to tend continuously to the one calculated at $F = 0$. What is more striking is that in a nonsupersymmetric theory the Lorentz violating Π_2 term also tends to zero as \tilde{k}^2/α' below the IR cut-off, reminiscent of the field theory behaviour with the naive Schwinger cut-off.

Consider nonplanar annulus amplitudes in bosonic string theory on a Dp -brane. As we shall see, the general structure of a one loop diagram can be very heuristically written as

$$A_{NP} \sim \int \frac{dt}{t} t^{-\frac{(p+1)}{2}} e^{-\tilde{k}^2/4t} f(t). \tag{2.1}$$

The function $f(t)$ includes kinematic factors as well as sums over all the open string states in the loop. The integration parameter t is the parameter describing the annulus. In the field theory limit $\alpha' \rightarrow 0$ we recover the expected nonplanar field theory contribution, with t playing the role of a Schwinger parameter. In addition all but the massless open strings (and in this case the tachyon whose contribution we discard) do not contribute in this limit. In the present discussion we are of course not interested in taking the field theory limit but will instead keep α' finite. The crucial feature of the amplitudes governing the IR behaviour is that the nonplanar integrands always come with a factor $e^{-\tilde{k}^2/4t}$ irrespective of whether we are above or below Λ_{IR} . When $\tilde{k}^2 \gg \alpha'$ the integrand is killed everywhere in the stringy region $t < \alpha'$ and the amplitude is close to the field theoretical result. Indeed one may make a large t expansion rendering the amplitude identical to the field theoretical one. On the other hand in the area of most interest below the IR cut-off we have $\tilde{k}^2 \ll \alpha'$ and hence stringy $t < \alpha'$ regions also contribute to the integral. If the integrand is finite and free of singularities then in the limit as $k \rightarrow 0$ the amplitudes clearly tend continuously to their commutative equivalents. Thus the finiteness of the string amplitudes immediately guarantees that the $k \rightarrow 0$ limits and the $\theta \rightarrow 0$ limits give the same physics. Moreover in this limit we may expand the $e^{-\tilde{k}^2/4t}$ factor inside the integral. The nett result is that far below the IR cut-off one-loop amplitudes may be written as

$$A(\theta, k) \sim A(0, k) \left(1 + \lambda \frac{\tilde{k}^2}{\alpha'} + \dots \right), \tag{2.2}$$

where λ is a factor including loop suppression and gauge couplings and the second piece is the leading term in the small \tilde{k}^2/α' expansion of the exponential factor. Note that the $A(0, k)$ prefactor includes the usual one-loop contributions of the commutative theory and hence all stringy threshold corrections. Thus although various compactification scenarios may result in vastly different threshold corrections, the leading effect of non-zero B field will always be of this form. (Extension to N -point amplitudes is trivial.)

Based on this generic expression for the amplitudes, phenomenology below Λ_{IR} takes on a characteristic form. First from the low energy point of view the net effect of the non-zero B field is simply to take the non-planar contribution to thresholds of gauge couplings and move them down to the IR cut-off, inserting between Λ_{IR} and Λ_{UV} a region approximating conventional noncommutative field theory. Below Λ_{IR} , the leading deviation from the commutative theory (including all its stringy thresholds) has a factor \tilde{k}^2/α' , with the dimensionality being made up by powers of α' .

Thus for example the Π_2 term is of the form

$$\Pi_2(k^2, \tilde{k}^2) \sim \lambda(\tilde{k}^2\alpha')^2 \tag{2.3}$$

in a nonsupersymmetric theory and

$$\Pi_2(k^2, \tilde{k}^2) \sim \lambda(\tilde{k}^2\alpha')^2 \frac{M_{SUSY}^2}{\alpha'} \tag{2.4}$$

in a theory with supersymmetry softly-broken at a scale M_{SUSY} . (Note that the factor of $(\tilde{k}^2\alpha')^2$ is simply to undo the power of \tilde{k}^{-4} in the above definition of Π_2 .) This introduces a birefringence into the trace-U(1) photon, a polarization dependent velocity shift of order

$$\Delta v \sim c \frac{\lambda M_{SUSY}^2 M_s^2}{M_{NC}^4}. \tag{2.5}$$

This effect is much milder than the naive expectation and can be made phenomenologically acceptable with a large M_{NC} even if the physical photon is predominantly made of trace U(1) photon as described in ref. [15]. The model dependent issue here which we will expand upon in the following sections is the coefficient λ which encapsulates the strength of the one-loop contributions (i.e. threshold corrections to couplings) relative to the tree level ones.

If the physical photon is decoupled from the trace U(1) photon (see for example [23] where the trace U(1) photon becomes weakly coupled in the IR and forms part of a hidden sector to break and mediate supersymmetry), then there can be interesting implications for gravity. Consider a theory where the physically observed Planck scale receives significant one-loop threshold corrections from the open string sector. This contribution can be computed from the two point function of the gravitons with the open string modes running in the loop. To gain some more intuition on what effects of noncommutativity might be, we turn to an effective field theory description. A reasonable (but, as it turns out, incorrect) guess for the effective field theory coupling the open string modes to the graviton is a lagrangian of the form

$$\mathcal{L} = \int d^4x \sqrt{-g} g^{\mu\mu'} g^{\nu\nu'} \frac{F_{\mu\nu} * F_{\mu'\nu'}}{4g^2}, \tag{2.6}$$

where there is, note, no star product between the "closed string metric" or in its determinant. The desired contribution can be computed from the two point function of the gravitons with the gauge bosons running in the loop, and thus our effective field theory above would generate "planar" and "non-planar" diagrams exactly as in the pure gauge case, the crucial point being the presence of a Moyal phase coming from the vertices. Thus

one might expect that in string theory, turning on a B -field would separate planar and non-planar contributions to the graviton two point function, in much the same way as for the photon. Thanks to UV/IR mixing the nonplanar contributions would change all the way down to Λ_{IR} below which they would asymptote to the values of the commutative theory. There, the leading deviation in Planck's constant from that of the purely commutative theory should be precisely as described above for the gauge couplings. As we will see in the section 5 the true picture is actually more subtle than this.¹ Nevertheless the effect we described persists; namely that subleading \tilde{k}^2/α' suppressed corrections in the two point function of the graviton lead to a modification of gravity at energy scales higher than Λ_{IR} .

3. UV/IR mixing in the bosonic string

We will first look at the 2-point function on the annulus for pure QED, equivalent to the noncommutative Yang-Mills action

$$S = - \int \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} \tag{3.1}$$

The contributions to the 2-point amplitudes on Dp -branes in a noncompact 26-dimensional volume requires open string vertex operators

$$V = g_{Dp} \epsilon_\mu \partial X^\mu e^{ik \cdot x}. \tag{3.2}$$

which have been appropriately normalised ($g_{Dp}^2 = (2\pi)^{p-2} g_c(\alpha')^{\frac{p-3}{2}}$). This gives the amplitude

$$A_2(k, -k) = -2\alpha' g_{Dp}^2 V_p \int_0^\infty dt (8\pi^2 \alpha' t)^{-\frac{(p+1)}{2}} \eta(it)^{-24} \times \int_0^t dx e^{-2\alpha' k \cdot G(x, x') \cdot k} (\epsilon_1 \cdot G_{xx'} \cdot \epsilon_2 - 2\alpha' (\epsilon_1 \cdot G_x \cdot k)(\epsilon_2 \cdot G_{x'} \cdot k)) \Big|_{x'=0}. \tag{3.3}$$

Here x, t play the role of *dimensionless* Feynman and Schwinger parameters respectively. At this point, we should comment that throughout this paper we shall take the fundamental domain of the annulus to be $[0, 1/2] \times [0, it]$.

Note that we write the measure with integration over all of the vertices, and then use the annulus' translation invariance to fix one vertex, including a volume factor of t . The one-loop Green's functions required depend on whether the diagram is planar or nonplanar, and are given by [21, 16, 24]

$$G^{\alpha\beta}(x, x') = I_0 \delta^{\alpha\beta} + J \frac{(\theta^2)^{\alpha\beta}}{\alpha'^2} + K \frac{\theta^{\alpha\beta}}{\alpha'}, \tag{3.4}$$

where, for the planar case,

$$I_0^P(x - x') = \log \left| t \frac{\theta_1\left(\frac{x-x'}{t}, \frac{i}{t}\right)}{\eta^3(i/t)} \right|, \quad J^P = 0, \quad K^P(x - x') = -\frac{i}{4} \epsilon(x - x'), \tag{3.5}$$

¹In the field theory limit, an effective vertex involving a graviton and two photons exists (and indeed we compute it), but there is no such simple Lagrangian from which it could be derived.

and for the nonplanar case,

$$I_0^{NP}(x-x') = \log t \frac{\theta_4\left(\frac{x-x'}{t}, \frac{i}{t}\right)}{\eta^3(i/t)}, \quad J^{NP} = \frac{-1}{8\pi t}, \quad K^{NP}(x+x') = \pm \frac{\pi}{t}(x+x'), \quad (3.6)$$

where the $+(-)$ in K^{NP} applies for the outer (inner) boundary. The feature of these expressions which ensures the regularization of the nonplanar diagram is the contraction $k.G.k$ appearing in the exponent of the integrand. We find

$$-2\alpha' k.G^P.k = -2\alpha' k^2 I_0^P, \quad (3.7)$$

$$-2\alpha' k.G^{NP}.k = -2\alpha' k^2 I_0^{NP} - \frac{\tilde{k}^2}{8\pi\alpha't}. \quad (3.8)$$

Having established the Green's functions, we can perform the integration by parts in equation (3.3) to extract the kinematics. Defining (for ease of notation) $\hat{\Pi}_2 = \Pi_2 \tilde{k}^{-4}$, we find

$$\begin{aligned} A_2^P &= \Pi_1^P(k^2, 0)[(\epsilon_1 \cdot \epsilon_2)k^2 - (\epsilon_1 \cdot k)(\epsilon_2 \cdot k)], \\ A_2^{NP} &= \Pi_1^{NP}(k^2, \tilde{k}^2)[(\epsilon_1 \cdot \epsilon_2)k^2 - (\epsilon_1 \cdot k)(\epsilon_2 \cdot k)] + \hat{\Pi}_2(k^2, \tilde{k}^2)[(\epsilon_1 \cdot \tilde{k})(\epsilon_2 \cdot \tilde{k})], \end{aligned} \quad (3.9)$$

where the standard gauge running is given by the formula

$$\Pi_1 = 4\alpha'^2 g_{D_p}^2 \int_0^\infty dt Z(t) e^{-\frac{\tilde{k}^2}{4\alpha't} \times a} \int_0^t dx e^{-2\alpha' k^2 I_0} (\dot{I}_0)^2, \quad (3.10)$$

while the Lorentz-violating piece is given by

$$\hat{\Pi}_2 = 4\pi^2 g_{D_p}^2 \int_0^\infty \frac{dt}{t^2} Z(t) e^{-\frac{\tilde{k}^2}{4\alpha't}} \int_0^t dx e^{-2\alpha' k^2 I_0^{NP}}. \quad (3.11)$$

In eq. (3.10) $a = 0$ or 1 for the planar or nonplanar case respectively. The term $Z(t)$ is the partition function for the model, which we shall take throughout to be

$$Z(t) = (8\pi^2 \alpha' t)^{-\frac{(p+1)}{2}} \eta(it)^{d-24}. \quad (3.12)$$

The parameter d is inserted to remove adjoint scalars from the theory: there are $p-1$ physical polarisations of the photon, and the remaining $25-p$ modes are scalars, so we can interpret the parameter d as removing d of these modes. This is performed either by considering string theory in $26-d$ dimensions [25, 21], or taking the spacetime to be, for example, $\mathbb{R}_{p+1} \times \mathbb{R}_{25-p-d} \times \mathbb{R}_d/Z_N$ with the p -brane at a singularity such that the orbifold projection removes the scalars [16]. In this way, we can alternatively consider d as modelling the effect of compactified dimensions, and hence we shall refer to d ‘compact dimensions’ throughout.

The forms (3.9)-(3.11) for the 1-loop amplitudes were already discussed in ref. [21] in the field theory limit. However it is important to note now that we have not taken any field theory limit and yet the \tilde{k} dependence is already entirely contained within the \tilde{k}^2 term: the sole effect of noncommutativity is to truncate the Schwinger integration to $2\pi t > \frac{\tilde{k}^2}{\alpha'}$, even in the full string expression.

Thus there are two regimes that we will consider. The first is the regime where $\tilde{k}^2/\alpha' \gg 1$. In this case the Schwinger integral is truncated to the region $2\pi t \gg 1$ and the integral is well approximated by the $t \rightarrow \infty$ limit. The second regime is where $\tilde{k}^2/\alpha' \ll 1$. In this case much of the Schwinger integral is over the region where $2\pi t \ll 1$ and one expects the $t \rightarrow \infty$ limit to be a poor approximation. In this limit a good approximation to the integral requires a modular transformation of the ϑ and η functions to the closed string channel. It is natural to think of α' as playing the role of the UV cut-off to the field theory, $\alpha' \equiv \Lambda_{UV}^{-2}$, and then this regime corresponds precisely to

$$k \ll \frac{M_{NC}^2}{\Lambda_{UV}} \equiv \Lambda_{IR}, \tag{3.13}$$

i.e. the region in the deep IR where the field theory computation breaks down.

Indeed the integral will become sensitive to the global structure of the compactified dimensions since the $t \rightarrow 0$ UV end of it corresponds to closed string modes in the deep IR. Note that [26, 27] studied the connection between IR poles and closed string tachyons; we shall neglect these as we are interested in extracting results relevant to consistent theories. There may be other thresholds as well as the string scale where for example winding modes of the compact dimensions start to contribute in the integral. In order for there to be an effective field theory description below Λ_{IR} these effects should add contributions independent of p . In order to incorporate these effects one can divide the Schwinger integral into regions $t \in [0, 1]$ and $t \in [1, \infty]$ where the two approximations are valid.

3.1 Brief review of planar diagrams

The methods for obtaining the low energy behaviour of string diagrams in order to derive the effective four-dimensional field theories have been well covered elsewhere [25, 28]. Since the integrals do not contain any evidence of the non-commutativity, as can be seen from the Green's functions, the only difference from the $B = 0$ calculation in this case is a phase dependence on the ordering of the vertices. The reader is referred to the appendix for details of the $t \rightarrow \infty$ limit for planar diagrams, which reviews some of the basic techniques that we will be using. We shall consider d dimensions transverse to the brane to be compactified with a radius close to or at the string scale, although we shall pay special attention to the case $d = 0$. The contributions to Π_1 and $\hat{\Pi}_2$ in this limit are given in equations (A.6) and (A.7).

The $t \rightarrow 0$ limit of the planar diagrams is the UV contribution, that is $t \in [0, 1]$ indicating momenta much higher than the string scale. It is well known as the planar contribution to the string threshold correction, however we present it here in order to emphasise the way that string theory is thought to render such contributions finite. Let us compute the $t \rightarrow 0$ contribution to the two point functions for a Dp -brane in 26 noncompact dimensions. We modular transform the expressions to give for the partition function

$$(8\pi^2\alpha't)^{-\frac{(p+1)}{2}} t^{12} e^{\frac{2\pi}{t}} (1 + 24e^{-\frac{2\pi}{t}} + \dots), \tag{3.14}$$

where, since we are assuming no compact dimensions, there are no winding modes, and thus

$$\Pi_{1UV}^P \rightarrow \frac{g_{D_p}^2}{16\pi^2(8\pi^2\alpha')^{\frac{(p-3)}{2}}} \int_0^1 dt t^{\frac{(21-p)}{2}} (2t)^{-2\alpha'k^2} \int_0^1 dy \left[24|\cos \pi y|^2 |\sin \pi y|^{-2-2\alpha'k^2} + 8|\cos \pi y|^2 |\sin \pi y|^{-2\alpha'k^2} \right]. \quad (3.15)$$

Here we cannot neglect the “pole” pieces, but perform the integral in terms of beta functions and analytically continue in the momentum, using

$$\int_0^1 dy |\cos \pi y|^a |\sin \pi y|^b = \frac{2}{\pi} B\left(\frac{a+1}{2}, \frac{b+1}{2}\right) \quad (3.16)$$

to give the zero-momentum limit

$$\Pi_{1UV}^P = \frac{5g_{D_p}^2}{2\pi^2(23-p)(8\pi^2\alpha')^{\frac{(p-3)}{2}}}, \quad (3.17)$$

a threshold contribution to the gauge couplings which is finite when $p < 23$.

Now, of course this computation is a cheat because it assumes that transverse space was noncompact. In a compact space, sooner or later in the $t \rightarrow 0$ limit we need to sum over winding sectors in the measure of the integral. Once the winding sectors are included the effective p is $p \equiv 25$ and the integral diverges. However this divergence is resolved in a way that is at least qualitatively well understood: the natural way to write A_{2UV} is with the parameter $S = \frac{\alpha'^2}{T}$ which reveals the expression to be in the $\alpha'k^2 \ll 1$ limit simply an IR pole due to a massless closed string tadpole. Indeed in this limit and when $p = 25$ the contribution from level n is proportional to

$$\int_0^{\alpha'} \frac{dT}{T^2} e^{-\frac{4\pi^2\alpha'}{T}(n-1)} = \int_{\alpha'}^{\infty} dS e^{-\frac{4\pi^2}{\alpha'}(n-1)S}, \quad (3.18)$$

as appropriate for closed string states with $m^2 = \frac{4\pi^2}{\alpha'}(n-1)$. Such tadpoles are a signal that we are expanding around the wrong vacuum, and the solution is to give a VEV to the relevant fields in order to remove them. In this way the background is modified by the presence of the tadpoles and the net effect is that systems with > 2 codimensions (i.e. $p < 23$) are insensitive to the moduli of the transverse dimensions, whereas those with 1 or 2 codimensions get threshold corrections that are respectively linearly or logarithmically dependent on the size of the transverse dimensions, but are still believed to be finite even when supersymmetry is broken by the construction. In principle in certain tachyon-free nonsupersymmetric cases one can resum the tadpole contributions to the tree-level perturbation series to achieve a finite result. The precise details are rather subtle and beyond the scope of this paper, and the reader is referred to refs. [29–31] for more details.

3.2 Non-planar diagrams in the $\tilde{k}^2/\alpha' \gg 1$ limit

We now turn to the non-planar diagrams. Once we turn on the B -field, the presence of the $e^{-\frac{\tilde{k}^2}{8\pi\alpha't}}$ regulating factor will cause the celebrated UV/IR mixing. We may treat the

UV and IR contribution at the same time in the two limits $\tilde{k}^2/\alpha' \ll 1$ and $\tilde{k}^2/\alpha' \gg 1$. Consider the second of these limits. The integrand is killed in the region where $t \ll \tilde{k}^2/\alpha'$ and hence we may always use the large t limit of the integrand. We obtain

$$\begin{aligned} \Pi_1 &= \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} \int_{2\pi\alpha'}^{\infty} dT T^{-\frac{(p-1)}{2}} \int_0^1 dy [(24-d)(1-2y)^2 - 8] e^{-Tk^2(y-y^2) - \frac{\tilde{k}^2}{4T}} \\ &\rightarrow \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} \int_0^1 dy [(24-d)(1-2y)^2 - 8] \left[\frac{4k^2 y(1-y)}{\tilde{k}^2} \right]^{\frac{(p-3)}{4}} K_{\frac{p-3}{2}}(\sqrt{y(1-y)k^2\tilde{k}^2}) \\ &\approx \begin{cases} \frac{d}{3} g_{D_p}^2 (4\pi)^{-\frac{p+1}{2}} \ln k^2 \tilde{k}^2, & p = 3, \\ \frac{d}{3} g_{D_p}^2 (4\pi)^{-\frac{p+1}{2}} 2^{p-5} \Gamma(\frac{p-3}{2}) |\tilde{k}|^{3-p}, & p > 3, \end{cases} \end{aligned} \quad (3.19)$$

where in the last step we assumed $|k||\tilde{k}| \ll 1$ or in other words momenta $|k| \ll M_{NC}$. In the case $p = 3$ this gives the same logarithmic running to a free field theory in the IR observed in the field theory. When $p > 3$ we find power law running in the IR as described in [32]. The Lorentz-violating term $\hat{\Pi}_2$ is given by

$$\begin{aligned} \hat{\Pi}_2 &= \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} \int_{2\pi\alpha'}^{\infty} dT T^{-\frac{(p+3)}{2}} \int_0^1 dy (24-d) e^{-Tk^2(y-y^2) - \frac{\tilde{k}^2}{4T}} \\ &\approx (24-d) \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} 2^{p-1} \Gamma(\frac{p+1}{2}) |\tilde{k}|^{-(p+1)} \end{aligned} \quad (3.20)$$

and shows a similar power law behaviour in the IR. For $p = 3$ and $d = 22$ we reproduce the result of [21]. This behaviour is entirely in line with what one would expect from the field theory.

3.3 Non-planar diagrams in the $\tilde{k}^2/\alpha' \ll 1$ limit

In this limit one expects to find behaviour differing from noncommutative field theory. We now have to split the integral into two halves, $t > 1$ and $t < 1$. The first IR part is treated similarly to the previous section, except in this case we simply set $\tilde{k} = 0$ in the integrand when we consider $\alpha' \rightarrow 0$, and should thus obtain the same results as in the planar case; it is straightforward to show that for $p > 3$

$$\begin{aligned} \Pi_{1IR}^{NP} &\approx \frac{d}{3} \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} \frac{2}{(p-3)} (2\pi\alpha')^{\frac{3-p}{2}}, \\ \hat{\Pi}_{2IR}^{NP} &\approx (24-d) \frac{g_{D_p}^2}{(4\pi)^{\frac{p+1}{2}}} \frac{2}{(p+1)} (2\pi\alpha')^{\frac{-(p+1)}{2}}. \end{aligned} \quad (3.21)$$

The contributions are roughly constant, and equal to those of the $\tilde{k}^2/\alpha' \gg 1$ limit when $\tilde{k}^2 = 4\alpha'$.

The second, UV, contribution for $t < 1$ is the most interesting, as it is this contribution which in field theory gives IR poles. We now modular-transform the expressions, and expand in powers of $e^{-\frac{2\pi}{t}}$. For no compact dimensions, we have

$$\Pi_{1UV}^{NP} \rightarrow \frac{g_{D_p}^2}{16\pi^2 (8\pi^2\alpha')^{\frac{(p-3)}{2}}} \int_0^1 dt t^{\frac{(21-p)}{2}} (2t)^{-2\alpha'k^2} e^{-\frac{\tilde{k}^2}{8\pi\alpha't}} e^{-\frac{\pi\alpha'k^2}{2t}} \int_0^1 dy \sin^2 2\pi y. \quad (3.22)$$

Note that for $\frac{\tilde{k}^2}{\alpha'} \rightarrow 0$ the integration is finite and the integral goes continuously to that of the commutative contribution, i.e. we have

$$\Pi_1^{NP}(\theta) = \Pi_1^{NP}(\theta = 0) \left(1 + \mathcal{O}\left(\frac{\tilde{k}^2}{\alpha'}\right) \right), \quad (3.23)$$

as promised in the Introduction; in other words, at momenta $k \ll \Lambda_{IR}$ the Wilsonian gauge couplings return to the values they would have had for a completely commutative theory with the same gauge group. Note that this statement is expected to be true even when $p \geq 23$ and in compact spaces for the following reason. In the finite examples we have seen, the effect of string theory is clearly to allow the limit $\tilde{k} \rightarrow 0$ to be taken continuously, and to give the same physics as $\theta = 0$. If this is true of any consistent UV completion, then it seems reasonable to assume that what's good for the planar diagrams is good for the nonplanar ones. In other words, if the diagrams are formally divergent, continuity demands that the vacuum shifts which remove the UV divergences (i.e. closed string tadpoles) in the $B = 0$ theory should do so in the $B \neq 0$ theory as well, upto $\mathcal{O}(\tilde{k}^2/\alpha')$ corrections. Note that this is true even though the non-planar diagrams do not factorise onto disks in the closed string channel; IR singularities arise from divergences in the partition function which are regulated by the $e^{-\frac{\tilde{k}^2}{8\pi\alpha't}} e^{-\frac{\pi\alpha'k^2}{2t}}$ term, and so when these divergences are cancelled, so are the IR poles.

This reasoning leads one to expect that the $\hat{\Pi}_2$ term is regulated, since it should tend to zero as $\theta \rightarrow 0$. Let us check this by computing the final contribution which is

$$\begin{aligned} \hat{\Pi}_{2UV}^{NP} &\rightarrow \frac{24g_{D_p}^2}{(8\pi^2\alpha')^{\frac{(p+1)}{2}}} \int_0^1 \frac{dt}{t^4} t^{\frac{(25-p)}{2}} t^{-2\alpha'k^2} e^{-\frac{\tilde{k}^2}{8\pi\alpha't}} e^{-\frac{\alpha'k^2\pi}{2t}} \int_0^1 dy \\ &\approx \frac{24}{19-p} \frac{g_{D_p}^2}{(8\pi^2\alpha')^{\frac{(p+1)}{2}}}. \end{aligned} \quad (3.24)$$

4. Supersymmetric models

To include the effects of worldsheet fermions we require the fermionic propagators [17]:

$$\langle \psi^\alpha(z_1) \psi^\beta(z_2) \rangle_\nu = G^{\alpha\beta} \frac{\theta_\nu(z_1 - z_2) \theta'_1(0)}{\theta_\nu(0) \theta_1(z_1 - z_2)}, \quad (4.1)$$

where the index ν specifies the spin structure, which must be summed over in the full amplitude. The above differs from the usual boundary fermion propagators purely by the replacement of the metric by the open string metric, but when we perform the rescaling of the external momenta and polarizations [21] it is transformed back to the standard propagators:

$$\begin{aligned} \langle \psi^\alpha(z_1) \psi^\beta(z_2) \rangle_\nu &\rightarrow \delta^{\alpha\beta} \frac{\theta_\nu(z_1 - z_2) \theta'_1(0)}{\theta_\nu(0) \theta_1(z_1 - z_2)} \\ &\equiv \delta^{\alpha\beta} G_\nu^\psi(z_1 - z_2), \end{aligned} \quad (4.2)$$

which we shall use from now on.

We wish to calculate the one-loop amplitude for two spacetime bosons with an arbitrary amount of supersymmetry in the loop, which is defined by the compact dimensions - and thus only affects the amplitude via the partition function. The vertex operators are

$$V^0 = g_{Dp} \epsilon_\mu (i\dot{X}^\mu + 2\alpha' k \cdot \psi \psi^\mu) e^{ik \cdot X}, \quad (4.3)$$

and the resulting amplitude gives

$$\Pi_1 = 4g_{Dp}^2 (\alpha')^2 \int_0^\infty dt \sum_\nu e^{2\alpha' \tilde{k}^2 J} Z_\nu(t) \int_0^t dx e^{-2\alpha' k^2 I_0} \left[(\dot{I}_0)^2 - (G_\nu^\psi(z(x)))^2 \right] \quad (4.4)$$

and

$$\hat{\Pi}_2 = 4\pi^2 g_{Dp}^2 \int_0^\infty dt \sum_\nu Z_\nu(t) e^{\frac{\tilde{k}^2}{4\alpha' t}} \int_0^t dx e^{-2\alpha' k^2 I_0}, \quad (4.5)$$

where $Z_\nu(t)$ is the partition function for the theory, and

$$\begin{aligned} z^P &= ix, \\ z^{NP} &= ix - 1/2. \end{aligned} \quad (4.6)$$

Thus the spacetime fermionic component does not contribute to the Lorentz-violating term, since the kinematics for it are just the standard commutative gauge pieces. The Lorentz-violating term thus derives from bosonic correlator exactly as in the bosonic string, the only difference being the partition function. Of course, if there is any supersymmetry then this term will vanish, as we expect, and the remaining Lorentz-preserving term can be calculated from the off-shell continuation of the fermionic piece. For $N \geq 1$ SUSY, Π_1 can be simplified using the identity

$$(G_\nu^\psi(z))^2 = \frac{\theta_\nu''(0)}{\theta_\nu(0)} - \partial^2 \log \theta_1(z) \quad (4.7)$$

to give

$$\Pi_1 = 4g_{Dp}^2 (\alpha')^2 \int_0^\infty dt \sum_\nu e^{2\alpha' \tilde{k}^2 J} Z_\nu(t) \frac{\theta_\nu''(0)}{\theta_\nu(0)} \int_0^t dx e^{-2\alpha' k^2 I_0}. \quad (4.8)$$

Again this is essentially the usual expression for computing threshold corrections, but with an exponential factor inserted for non-planar diagrams.

To summarize the results of this and the previous two sections, as advertised in the Introduction, both bosonic and supersymmetric theories are found to tend continuously to the $B = 0$ theory as $k \rightarrow 0$. In particular the couplings freeze out below Λ_{IR} and the entire region above Λ_{IR} can now be consistently integrated out in the usual Wilsonian manner. The phenomenological footprint of the non-zero B field is then in the dispersion relation of massless particles, and in particular a birefringence of the trace-U(1) photon, which gets a polarization dependent velocity shift of order

$$\Delta v \sim c \frac{M_{\text{SUSY}}^2 M_s^2}{M_{NC}^4}. \quad (4.9)$$

Whether the EM photon feels this effect is a model dependent question.

5. The two point function of the graviton

We now turn to the effect of the non-zero B field on gravity by focussing on the graviton two-point function. In particular, consider the corrections to the Newtonian force law of gravity due to the coupling of the graviton to gauge fields at one-loop. The momentum dependence of these corrections determines the running of Planck's constant, and our experience with gauge couplings suggests that this also may be subject to UV/IR mixing.

In the naive extension of noncommutative field theory of eq. (2.6), the one loop contributions divide into planar and non-planar exactly as they do for the trace $U(1)$ photon. However in string theory the relevant diagram is an annulus with two graviton (closed string) vertices on the interior of the world sheet, and so the only way that planar could be distinguished from nonplanar would be either for there to be some kind of radial ordering effect in the vertices, or for there to be a limit in which the major contribution to the diagrams came from when the vertices were on the edges of the annulus. Neither of these possibilities is true and so, even before making any computation, it seems unlikely that there will be a simple field theory approximation involving Moyal products. The field theory limit has been the subject of a recent study in ref. [33] where it was indeed found to be a rather complicated issue. However for the present study we do not need to derive the effective action (and indeed we don't): we will instead examine the modification of the Newtonian force law between matter (open string) fields on the brane, by looking at the two point function determined at the string theory level.

By restricting our attention to the force law between matter fields, we are evading a significant technical difficulty, namely that in a sense we have two metrics, one for open strings and one for closed. We wish to examine the momentum dependence of the gravitational force between *open strings* confined to magnetised D -branes. In principle we ought to be doing this by factorizing a four point open string amplitude on the graviton two point function. The relations for G, g and θ imply

$$g^{\mu\nu} = G^{\mu\nu} - \frac{(\theta G \theta)^{\mu\nu}}{4\pi^2(\alpha')^2}. \tag{5.1}$$

determining the coupling of the matter on the brane to gravity. We must choose a coordinate system where the components of the metric $g_{\mu\nu}$ are made small ($\alpha' F_{\mu\nu}$) for the dimensions in which magnetic field is turned on, so that the noncommutativity tensor $\theta^{\mu\nu}$ can be tuned to the desired values. Then the relevant momentum scale for the amplitude is given by the Mandelstam variables running through the loop, determined from the external momenta *as contracted with the open string metric*. Importantly the closed string metric is vastly different in the regimes of interest: for the exchange of a graviton with four-momentum q_μ between open strings, the Mandelstam variables correspond to scales of order

$$q^2 \equiv q_\mu q_\nu G^{\mu\nu}; \tag{5.2}$$

but if we were interested in graviton exchange between external graviton states, it would be more appropriate to use

$$q_\mu q_\nu g^{\mu\nu} = q^2 - \frac{\tilde{q}^2}{4\pi^2(\alpha')^2}, \tag{5.3}$$

which, by definition, for $q \sim \Lambda_{IR}$, would be of order $\sim M_s^2$. In the former case, as we are only dealing with graviton propagators the difference is immaterial since we can always rescale the graviton states to absorb the difference, but the correct procedure for (or indeed physical meaning of) the latter is less clear. (For example we would probably want more information about the other contributions in such a process coming from B fields, and also more information about what the asymptotic states are – i.e. the effective field theory.) Thus in calculating the graviton correlator, we shall decompose the square of the momentum in terms of the open string quantities, and consider $\tilde{k}^2/\alpha' \gg \alpha'k^2$, while $k^2\tilde{k}^2 \ll 1$.

We shall restrict the discussion to exchanges between $D3$ -brane states, for which (since there can be no orientifold planes) we need only consider the annulus for the induced gravity on the brane at one loop. A typical model for this scenario would be $D3/D7$ -branes at a C^6/\mathbb{Z}_N orbifold singularity [34], with a magnetic flux on the 1 and 2 directions. However, we shall keep the discussion as general as possible. We proceed initially as in ref. [35, 36] to extract the correction to $\frac{M_{Pl}^2}{16\pi}$, denoted δ , by considering the following kinematic portion of the graviton two-point function:

$$\langle V_G(h^1, k)V_G(h^2, -k) \rangle \supset -\frac{\delta}{4} h_{\mu\nu}^1 h_{\lambda\rho}^2 g^{\mu\lambda} k^\nu k^\rho \equiv A_\delta \tag{5.4}$$

where the vertex operators are given by

$$V_G(h, k) = g_s^2 N_G \frac{2}{\alpha'} h_{\mu\nu} (\partial X^\mu(z) - \frac{i\alpha'}{2} : k \cdot \psi \psi^\mu(z) :) (\bar{\partial} X^\nu(\bar{z}) - \frac{i\alpha'}{2} : k \cdot \tilde{\psi} \tilde{\psi}^\nu(\bar{z}) :) e^{ik \cdot X(z, \bar{z})}. \tag{5.5}$$

In the open string channel the Green's function is given by modular transforming the result of ref. [17]:²

$$\mathcal{G}^{\mu\nu}(w_1, w_2) = -\frac{\alpha'}{2} I^C G^{\mu\nu} + J^C \frac{(\theta^{\mu\alpha} G_{\alpha\beta} \theta^{\beta\nu})}{8\pi^2 \alpha'} - K^C \frac{\theta^{\mu\nu}}{2\pi}, \tag{5.6}$$

where

$$\begin{aligned} I^C &= \ln \left| \frac{\theta_1(w_1 - w_2, it) \theta_1(w_1 + \bar{w}_2, it)}{4\pi^2 \eta^6(it)} \right|^2 - \frac{4\pi}{t} |\Im(w_1 - w_2)|^2, \\ J^C &= \ln \left| \frac{\theta_1(w_1 - w_2, it)}{\theta_1(w_1 + \bar{w}_2, it)} \right|^2 - \frac{4\pi}{t} \left(|\Re(w_1 + \bar{w}_2)|^2 - \Re(w_1) - \Re(w_2) \right), \\ K^C &= \ln \theta_1(w_1 + \bar{w}_2, it) - \ln \theta_1(\bar{w}_1 + w_2, it) - \frac{2\pi i}{t} \Im \left((w_1 + \bar{w}_2 + 1/2)^2 \right) - 2\pi i f(\Im(w_1 - w_2)) \end{aligned} \tag{5.7}$$

²Note that this choice of propagator differs slightly from those given elsewhere [16, 21], but it was asserted in [17] that the additional terms are necessary to ensure periodicity and obedience to the equations of motion. They cause a discrepancy when the fields are taken to the boundary; (3.5, 3.6) are not obtained from (5.7). However, the closed string propagators only differ by linear terms in J^C and K^C , plus the function f which plays no essential role in amplitudes (merely ensuring that the derivatives of the logs in the antisymmetric portion contain no discontinuities). The reader can check that (5.13) is unchanged by these, and since I^C is identical for both versions, so are all the other results in this section.

and where $f(x) \equiv -[x/t]$, $[y]$ denotes the closest integer to y . Thus the self-contraction terms, with normal-ordering and the $w_1 \rightarrow w_2$ limit performed, are

$$C^{\mu\nu}(w, \bar{w}) = -\left(\frac{\alpha'}{2}G^{\mu\nu} + \frac{(\theta^{\mu\alpha}G_{\alpha\beta}\theta^{\beta\nu})}{8\pi^2\alpha'}\right) \ln \left| \frac{\theta_1(w + \bar{w}, it)}{2\pi\eta^3(it)} \right|^2 - \frac{(\theta^{\mu\alpha}G_{\alpha\beta}\theta^{\beta\nu})}{8\pi^2\alpha'} \frac{8\pi}{t} (2\Re^2(w) - \Re(w)). \quad (5.8)$$

The fermionic Green's functions are obtained from the torus functions using the doubling trick:

$$\psi^\mu(w) = \begin{cases} \psi^\mu(w), & \Re(w) > 0, \\ i\left(\frac{g+F}{g-F}\right)_\nu^\mu \tilde{\psi}^\nu(-\bar{w}), & \Re(w) < 0. \end{cases} \quad (5.9)$$

We obtain

$$\begin{aligned} \langle \psi^\alpha(z)\psi^\beta(w) \rangle_\nu &= g^{\alpha\beta} G_\nu^\psi(z-w), \\ \langle \tilde{\psi}^\alpha(\bar{z})\tilde{\psi}^\beta(\bar{w}) \rangle_\nu &= g^{\alpha\beta} G_\nu^\psi(\bar{z}-\bar{w}), \\ \langle \psi^\alpha(z)\tilde{\psi}^\beta(\bar{w}) \rangle_\nu &= -i \left(g^{\alpha\beta} + 2\frac{(\theta G\theta)^{\alpha\beta}}{4\pi^2(\alpha')^2} - 2\frac{\theta^{\alpha\beta}}{2\pi\alpha'} \right) G_\nu^\psi(z+\bar{w}). \end{aligned} \quad (5.10)$$

As for the gauge bosons, the physical behaviour naturally splits into long distance $\tilde{k}^2/\alpha' \ll 1$ and short distance $\tilde{k}^2/\alpha' \gg 1$ regimes. In the former, gravity will be dominated by the low energy modes, for which the usual corrections to Planck's constant apply. We can expand the amplitude as a power series in k^2 and \tilde{k}^2 , and neglect the terms $O(k^2)$ relative to $O(\tilde{k}^2)$. In the short distance regime however such an expansion is no longer appropriate, but the amplitude still has terms with a prefactor of \tilde{k}^2 which we should consider dominating over those prefixed by k^2 . In this way, we may consider the same correlators as being typical dominating terms in the amplitude for the non-zero B -field corrections to both limits; one such term is

$$A \supset \int_0^\infty dt \int d^2z \int d^2w -g_s^2 \frac{\alpha'}{2} N_G^2 \langle \partial X^\mu(z) \bar{\partial} X^\lambda(\bar{w}) e^{ik \cdot X(z, \bar{z})} e^{-ik \cdot X(w, \bar{w})} \rangle \langle k \cdot \tilde{\psi} \tilde{\psi}^\nu(\bar{z}) k \cdot \psi \psi^\rho(w) \rangle \quad (5.11)$$

which has a leading contribution of the form

$$\int_0^\infty dt \int d^2z \int d^2w g_s^2 N_G^2 \frac{\tilde{k}^2}{4\pi^2\alpha'} Z(t) (G_\nu^\psi)^2 k_a k_b \partial_z \mathcal{G}^{a\mu} \bar{\partial}_w \mathcal{G}^{b\lambda} \langle e^{ik \cdot X(z, \bar{z})} e^{-ik \cdot X(w, \bar{w})} \rangle \equiv L_\delta + \dots \quad (5.12)$$

Here L_δ is the component of this term which contributes to A_δ , and we have included in the partition function the Chan-Paton summation, which corresponds to summing over the Casimirs of the representations of the gauge group. We shall leave a complete analysis to future work, and consider the contribution from the corner of the moduli space where $t > 1$. Here we can take the derivatives of the Green's functions to be given by the leading order terms as $t \rightarrow \infty$ - as for the gauge theory case, this is equivalent to a field theory calculation, but it is more expedient to perform the calculation from string theory. We find

that the behaviour is dominated by the correlator of the exponentials: this is given by

$$\begin{aligned}
 \langle e^{ik \cdot X(z_1)} e^{-ik \cdot X(z_2)} \rangle &= \left| \frac{\theta_1(z-w, it)}{2\pi\eta^3(it)} \right|^{-2\alpha'k^2 - \frac{\tilde{k}^2}{4\pi^2\alpha'}} \left| \frac{\theta_1(z+\bar{w}, it)}{2\pi\eta^3(it)} \right|^{-2\alpha'k^2 + \frac{\tilde{k}^2}{4\pi^2\alpha'}} \\
 &\quad \left| \frac{\theta_1(z+\bar{z}, it)}{2\pi\eta^3(it)} \right|^{\alpha'k^2 + \frac{\tilde{k}^2}{8\pi^2\alpha'}} \left| \frac{\theta_1(w+\bar{w}, it)}{2\pi\eta^3(it)} \right|^{\alpha'k^2 + \frac{\tilde{k}^2}{8\pi^2\alpha'}} \\
 &\quad \exp \left[\frac{4\pi\alpha'k^2}{t} |\Im(z-w)|^2 \right] \exp \left[\frac{-\tilde{k}^2}{\pi\alpha't} |\Re(z-w)|^2 \right]. \quad (5.13)
 \end{aligned}$$

Note that this correctly factorises onto the corresponding boundary amplitude. To take the field theory limit now, we write $T = \pi\alpha't$, $y = \Im(z)/t$, use the translation invariance of the annulus to fix $\Im(w) = 0$, and write $\Re(z) = x$, $\Re(w) = x'$, and insert the partition function and kinematic factors, with a sum over spin structures. Making use of the identity (4.7) and assuming $N \geq 1$ supersymmetry, so that after multiplying by the partition function all spin-structure-independent terms vanish, we obtain the prefactor

$$F(T) \equiv (8\pi T)^{-2} \sum_{\nu} Z_{\nu} \left(\frac{T}{\pi\alpha'} \right) \frac{\theta_{\nu}''(0)}{\theta_{\nu}(0)}. \quad (5.14)$$

We shall assume $F(T)$ has the behaviour

$$\lim_{T \rightarrow \infty} T^2 F(T) = \beta, \quad (5.15)$$

where β is a constant. If we now insert the factors from our “typical” contribution, we obtain

$$\begin{aligned}
 L_{\delta}^{FT} &= g_s^2 N_G^2 \frac{\tilde{k}^2}{4\pi^2\alpha'} \int_{\pi\alpha'}^{\infty} dT T F(T) \int_0^1 dy (1-2y)^2 e^{-4k^2 T y(1-y)} \\
 &\quad \int_0^{1/2} dx \int_0^{1/2} dx' e^{-\frac{\tilde{k}^2}{T}(x-x')^2} |\sin 2\pi x \sin 2\pi x'|^{\alpha'k^2 + \frac{\tilde{k}^2}{8\pi^2\alpha'}}. \quad (5.16)
 \end{aligned}$$

As discussed, in contrast to a noncommutative field theory, there is no separation into planar and non-planar diagrams.

Since we are considering the regime $k^2 \tilde{k}^2 \ll 1$, we reorder the integration and use the leading behaviour of the Bessel function K_0 as in ref. [21] to give

$$\begin{aligned}
 L_{\delta}^{FT} &\approx g_s^2 N_G^2 \frac{\tilde{k}^2}{4\pi^2\alpha'} \frac{\beta}{6\pi} \log |k^2 \tilde{k}^2| B^2 \left(\frac{1}{2}, \frac{1}{2} + \frac{\alpha'k^2}{2} + \frac{\tilde{k}^2}{16\pi^2\alpha'} \right) \\
 &\approx g_s^2 N_G^2 \frac{\tilde{k}^2}{4\pi^2\alpha'} \frac{\beta}{6\pi} \frac{16\pi^3\alpha'}{\tilde{k}^2} \log |k^2 \tilde{k}^2| \quad (5.17)
 \end{aligned}$$

and thus this contribution to the graviton renormalisation, after we include $N_G = (8\pi G_4)^{1/2}/2\pi$ (where G_4 is Newton’s constant) is given by

$$\delta \supset -\frac{4g_s^2\beta G_4}{3\pi} \log |k^2 \tilde{k}^2|. \quad (5.18)$$

Note that when we sum over all equivalent diagrams and thus remove the field theory singularity, the $\log \tilde{k}^2$ term still remains. However, this is of course not a singularity, as we have up to this point been considering $\tilde{k} \gg \alpha'$.

As \tilde{k} decreases, the amplitude should smoothly revert to the correction for $\theta = 0$. To find the deviation from Newtonian behaviour at large distances we are interested in the variation of δ for small \tilde{k}^2/α' , which as discussed above will be dominated by the same terms as in the large limit; for the term we have been considering we obtain

$$\begin{aligned}
 L_{\delta}^{FT} &= g_s^2 N_G^2 \frac{\beta \tilde{k}^2}{16\pi^2 \alpha'} \int_{\pi\alpha'}^{\infty} \frac{dT}{T} \int_0^1 dy (1-2y)^2 e^{-4k^2 T y(1-y)} + O\left(\left(\frac{\tilde{k}^2}{4\pi^2 \alpha'}\right)^2\right) \\
 &= \frac{g_s^2 \beta \tilde{k}^2 G_4}{8\pi^3 \alpha'} 2 \int_0^1 dy y(1-y)(1 - \gamma_E - \log 4\pi - \log(\alpha' k^2 y(1-y))) + O((\alpha' k^2)^2)
 \end{aligned}
 \tag{5.19}$$

from which we extract the contribution to the renormalisation:

$$\delta \supset -\frac{g_s^2 \beta G_4 \tilde{k}^2}{24\pi^3 \alpha'} \left(-\frac{5}{3} + \gamma_E + \log 4\pi + \log \alpha' k^2 \right).
 \tag{5.20}$$

6. Phenomenology: modification of gravity at a mm

We now turn to phenomenological issues beginning briefly with the possibility of Lorentz violation in the photon. In the Introduction we mentioned birefringence of the trace U(1) photon which is constrained by astrophysical observations. Taking into account our analysis and the fact that the Lorentz violating operator Π_2 vanishes in a fully supersymmetric theory, the velocity shift is of order

$$\Delta v \sim c \frac{\lambda M_{\text{SUSY}}^2 M_s^2}{M_{NC}^4}.
 \tag{6.1}$$

Following ref. [37, 38] a relatively firm constraint comes from “time of flight” signals from pulsars;

$$\frac{\sqrt{\lambda} M_{\text{SUSY}} M_s}{M_{NC}^2} \sim \sqrt{\lambda} \frac{M_{\text{SUSY}}}{\Lambda_{IR}} < 2 \times 10^{-8},
 \tag{6.2}$$

where λ is here a measure of the one-loop suppression in the gauge diagrams, and M_{SUSY} is a measure of the supersymmetry breaking. A natural question to ask is how low the IR cut-off can be; in other words, *is it likely that a regime that is well approximated by noncommutative gauge theory will ever be accessible?* Alas, the answer is no. Since λ is a loop suppression factor involving known gauge couplings it will be at least of order 10^{-3} assuming that the mixing between the physical photon and trace U(1) photon is of order unity. However supersymmetry is broken and transmitted, one should almost certainly take $M_{\text{SUSY}} > 1\text{TeV}$ giving

$$\Lambda_{IR} > 10^9 \text{GeV}.
 \tag{6.3}$$

This bound is comparable to those coming from atomic physics calculated in ref. [39];

$$M_{NC} > 10^{14} \text{GeV}.
 \tag{6.4}$$

Assuming that $M_s < M_{Pl}$, that bound translates into

$$\Lambda_{IR} > 2 \times 10^{10} GeV. \tag{6.5}$$

If the physical photon has significant mixing with the trace U(1) photon, it seems likely therefore that a non-zero B field would be felt as residual Lorentz violation rather than full blown noncommutative field theory. For more detailed discussion of these questions see ref. [15].

Consider instead the possibility that the physical photon does not mix with the trace U(1) photon. This could be the case if the trace U(1) photon forms part of a hidden sector, or if the trace U(1) is spontaneously broken by for example a Fayet-Iliopoulos term, if it is anomalous. In this case M_{NC} can be much lower and a significant effect can show up in gravitational interactions. Our general analysis shows that the graviton two point function in a theory with nonvanishing θ tends continuously to the commutative one with leading terms suppressed by factors of \tilde{k}^2/α' . Neglecting the possible implications of a non-trivial tensor structure for the moment, the mildest effect one expects is a modification of the Newtonian force law which derives from it. The observable effects will make themselves felt as we probe the gravitational interaction at shorter distances. As we saw, there is something akin to a “nonplanar” one-loop contribution in the sense that $\tilde{G}(\mathbf{k})$ interpolates between the $\tilde{k}^2 \gg \alpha'$ regime and the $\tilde{k}^2 \ll \alpha'$ regime where it deviates from the purely commutative model as \tilde{k}^2/α' . Neglecting tensor structure, we can therefore model the two point function as

$$\tilde{G}(\mathbf{k}) = \frac{1}{M_{Pl}^2 \mathbf{k}^2} \frac{1 + f(\frac{\tilde{k}^2}{\alpha'})}{1 + \lambda} \tag{6.6}$$

where $f(x) \rightarrow \lambda(1 + \mathcal{O}(x))$ for $x \ll 1$ and tends to the short range behaviour for $x \gg 1$. Here M_{Pl}^2 is the one loop Planck mass, which includes also tree level disk diagram contributions such as those considered in ref. [33]. For example if we assume that the one-loop contribution has power law behaviour $\sim |\tilde{k}|^{(3-p)}$ we can model the total tree and one-loop two point function as

$$\tilde{G}(\mathbf{k}) = \frac{1}{M_{Pl}^2 \mathbf{k}^2} \frac{1}{(1 + \lambda)} \left(1 + \lambda \left(\frac{1}{1 + \frac{\mathbf{k}^2}{\alpha'}} \right)^{\frac{p-3}{2}} \right). \tag{6.7}$$

The coefficient λ encapsulates the one-loop open string contribution to Planck’s constant in the commutative theory with $\theta = 0$, which can be significant and is model dependent. Indeed there are generic scenaria that lead to the extremes $\lambda \gg 1$ and $\lambda \ll 1$:

1. The ADD scenario [41, 42]: the Standard Model is associated with a local brane configuration (for example in a “bottom-up” construction as per the previous section), with the 4D Einstein-Hilbert action deriving from the dimensionally reduced 10D action. In this case the one loop correction will be localized whereas the large tree-level M_{Pl}^2 is the result of a large volume. The one loop open string contribution will therefore be suppressed by a factor

$$\lambda \sim \frac{1}{V_{10-p}} \tag{6.8}$$

where V_{10-p} is the extra-dimensional volume in units of $\sqrt{\alpha'}$. 3-branes in the original ADD scenario with TeV scale gravity would therefore lead to a tiny λ , but one could imagine the Standard Model localized on wrapped D7-branes for example, in which case intermediate values of λ are possible.

2. The DGP scenario [43]: gravity is localized to a 3-brane in infinite or large extra dimensions by one-loop diagrams with matter (brane) states in the loop. The novel feature is that gravity becomes higher dimensional at long distances, offering an explanation of the observed cosmological acceleration. In this case one expects $\lambda \gg 1$ in the region where gravity is 4 dimensional. In more detail, the full action consists of a bulk term and a one loop induced brane term;

$$M_{Pl}^2 \left(\int d^4x \sqrt{g_4} R^{(4)} + \rho_c^{D-4} \int d^Dx \sqrt{g_D} R^{(D)} \right), \quad (6.9)$$

where $R^{(4)}$ is the curvature form the induced metric on the brane. Since ρ_c^{D-4} appears in the propagator with a factor k^2 it is natural that the cross-over length scale *above* which gravity appears D dimensional, generically given by [44]

$$R_c = \alpha'^{\frac{6-D}{4}} \frac{\rho_c^{\frac{D-4}{2}}}{\rho_c^{\frac{D-4}{2}}}. \quad (6.10)$$

This possibility has been analyzed for (Type I) open string models in ref. [35, 36, 44], where in practice a number of different threshold effects are possible if the matter branes wrap some compact internal dimensions. The precise details of these other thresholds will not change our conclusions about the effect of UV/IR mixing.

To see the effect of the one-loop corrections on the potential between two point particles consider for example $\theta^{12} = \theta$. In this case

$$\tilde{\mathbf{k}}^2 = \theta^2(\mathbf{k}_1^2 + \mathbf{k}_2^2) = \theta^2 \mathbf{k}^2 \sin^2 \vartheta, \quad (6.11)$$

where ϑ is the angle to the 3 direction. The potential depends on the angle ϑ and is given by the retarded Green's function;

$$\begin{aligned} V(\mathbf{x}) &= \int dt G_R(t, \mathbf{x}) \\ &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} \tilde{G}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \end{aligned} \quad (6.12)$$

which leads to

$$V(r, \vartheta) = \frac{1}{8\pi M_{Pl}^2 r} \left(1 + \frac{1}{1+\lambda} \int_0^\infty \left(f\left(\frac{r_c^2}{r^2} y^2\right) - \lambda \right) e^{-y \cos \vartheta} J_0(y \sin \vartheta) dy \right) \quad (6.13)$$

where

$$r_c = \frac{\theta}{\sqrt{\alpha'}} = \frac{M_s}{M_{NC}^2}. \quad (6.14)$$

In the limit where $r \cos \vartheta \gg r_c$ we may expand f inside the integral. Using the identity

$$\int_0^\infty y^{m \geq 0} e^{-y \cos \vartheta} J_0(y \sin \vartheta) dy = (-1)^m m! P_m(\cos \vartheta) \quad (6.15)$$

we find that the leading deviation from Newtonian behaviour is a quadrupole moment that sets in at $r \sim r_c$: indeed if $f(x) = \lambda(1 + \beta x + \dots)$ we find

$$V(r, \vartheta) = \frac{1}{8\pi M_{Pl}^2 r} \left(1 + \frac{\lambda\beta(3 \cos^2 \vartheta - 1)}{(1 + \lambda)} \frac{r_c^2}{r^2} + \mathcal{O}\left(\frac{r_c^4}{r^4}\right) \right). \quad (6.16)$$

The radius r_c is the distance above which Planck's constant tends to the $B = 0$ one-loop value. This is a potential which can be compared directly with the experimental bounds presented in ref. [45]. Also note that there is a direction given by $\cos \vartheta = 0$ where the physics is identical to $\theta = 0$ physics.

At smaller distances the “nonplanar” contribution to Planck's constant diminishes. For $r \ll r_c$ we may use the identity

$$e^{-y \cos \vartheta} J_0(y \sin \vartheta) = \sum_{n=0}^\infty \frac{(-1)^n y^n}{n!} P_n(\cos \vartheta) \quad (6.17)$$

and approximate

$$f\left(\frac{r_c^2}{r^2} y^2\right) = \lambda \left(\frac{1}{1 + \frac{r_c^2}{r^2} y^2} \right)^{\frac{p-3}{2}} \quad (6.18)$$

to find the first few harmonics as

$$V(r, \vartheta) = \frac{1}{8\pi M_{Pl}^2 r} \left(\frac{1}{1 + \lambda} + \sum_{n=0}^{p-4} \frac{(-1)^n B\left(\frac{p-n-4}{2}, \frac{n+1}{2}\right)}{2n!} P_n(\cos \vartheta) \left(\frac{r}{r_c}\right)^{1+n} + \mathcal{O}\left(\frac{r}{r_c}\right)^{p-2} \right). \quad (6.19)$$

The leading term is the tree-level Planck's constant, and the subleading terms grow with radius, as they should, to build up the full one-loop Planck's constant at large distance.

The most notable general conclusion from this analysis is simply that the distance scale at which the modification of gravity takes place,

$$r_c = \frac{\theta}{\sqrt{\alpha'}} = \frac{M_s}{M_{NC}^2}, \quad (6.20)$$

can be much larger than the inherit distance scales in the model. For example if $M_s \sim M_{Pl}$ and $M_{NC} \sim 1 TeV$ then $r_c \sim 1 mm$ (the same numerical coincidence as the large extra dimension scenarios with 2 extra dimensions).

7. Conclusions

Noncommutative field theory provides a theoretical framework to discuss effects of non-locality and Lorentz symmetry violation. Proper understanding and better control of the

UV/IR mixing has been a serious obstacle for the field theory. In this paper, we have emphasised that the IR singularities are just a reflection of the fact that field theory is UV divergent. Consequently any attempt to resolve them without modifying the UV behaviour of the field theory is doomed, and they can only be consistently smoothed out in a UV finite theory. We have demonstrated this explicitly by considering noncommutative field theory as an approximation to open string theory with a background B -field. We showed that the noncommutative field theory description is valid only for the intermediate range of energy scale $\Lambda_{IR}^2 \equiv \alpha' M_{NC}^2 < k^2 < 1/\alpha'$ and explored what happens outside this range. The IR singularities are rendered harmless and in fact, long before they are reached, the singular IR physics of the noncommutative theory is replaced by regular physics that is dictated by the UV finiteness of strings. In many non-supersymmetric theories, tachyonic instabilities arise from the modified dispersion relation (1.4) [46], which our analysis implies are also resolved by embedding into an UV-complete theory, as discussed in the context of field theory in [15].

With the UV/IR mixing under control, one can now reliably study how noncommutative geometry modifies the IR physics. Below the noncommutative IR scale Λ_{IR} , normal Wilsonian behaviour is resumed and the low energy physics can be described in terms of ordinary local physics with residual Lorentz violating operators. Indeed the theory tends continuously to the commutative $B = 0$ field theory, with the Lorentz violating operators remaining as a footprint in the low energy phenomenology of the string scale physics. A second important example of how the low energy physics is modified arises in the gravitational sector. We studied how the noncommutative geometry may modify gravity by considering the graviton two point function. The departure from the ordinary Newtonian potential can be much more significant and happen at much lower energy scales than those suggested by any extra dimensions.

One aspect of the present study that requires further elaboration is the nature of the effective field theory in the gravity sector and the resulting cosmology. Because of the difficulty of extracting an effective field theory for the gravitational sector it is not clear how these features will turn out, or indeed if they lead to any strong observational constraints.

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A. Field theory limits of string diagrams - a review

First we divide the Schwinger integrals as described above so that

$$\Pi_1(k, -k) = \Pi_{1IR} + \Pi_{1UV}, \tag{A.1}$$

where UV and IR indicate $t \in [0, 1]$ and $t \in [1, \infty]$ respectively. Considering the IR contribution of the planar diagram note that, if we reduce $\theta \rightarrow 0$, then the field-theory

limit should be the same for planar and non-planar diagrams; equivalently, they should be the same up to $O(\alpha')$ corrections. This is not immediately obvious from the Green's functions, but we must bear in mind that the planar diagrams have spurious poles on the worldsheet, and the string amplitude is strictly only defined after analytically continuing the momentum [25, 28]. The field theory limit is obtained by taking $t \gg 1$ and excising the regions around the poles - i.e. the region $|x - x'| < 1$ and $t - |x - x'| < 1$ - and then keeping terms of lowest order in w :

$$I_0^{P,NP} = -\frac{(x - x')^2}{t} + \pi|x - x'| \pm \Delta + O(e^{-2\pi t}), \tag{A.2}$$

where Δ is of order w , and the $-(+)$ preceding it applies to planar (non-planar) diagrams. We retain this term due to the presence of the tachyon, as in [25, 21]; it is given by

$$\Delta = e^{-2\pi x} + e^{2\pi(x-t)} \tag{A.3}$$

so that $\dot{\Delta}^2 = -4\pi^2 + O(w)$, but for the superstring we shall find that it is irrelevant. Inserting the above into (3.10) and extracting the contribution from the first level in the loop, we find

$$\begin{aligned} \Pi_{1IR}^P(k^2) &= -4\alpha'^2 g_{D_p}^2 \int_1^\infty dt (8\pi^2 \alpha' t)^{-\frac{(p+1)}{2}} e^{2\pi t} (1 + (24 - d)e^{-2\pi t} + \dots) \times \\ &\quad \int_0^t dx e^{-2\alpha' k^2 \pi(x - \frac{x^2}{t})} \left[\frac{-2\pi x}{t} + \pi + \dot{\Delta} + \dots \right]^2 \\ &= \frac{-g_{D_p}^2}{(4\pi)^{\frac{(p+1)}{2}}} \int_{2\pi\alpha'}^\infty dT T^{-\frac{(p-1)}{2}} \int_0^1 dy e^{-Tk^2(y-y^2)} [(24-d)(1-2y)^2 - 8 + \dots]. \end{aligned} \tag{A.4}$$

This result looks just like the field theoretical Schwinger integral as it should (note the change to the parameters $T = 2\pi\alpha't$ and $y = x/t$). We have not explicitly written the tachyonic contribution or contributions coming from states at higher excitation level: the tachyon because it is unphysical, and the higher states because their nonplanar counterparts in the IR ($p \rightarrow 0$) are all finite. For the moment we need only note that a contribution at level n yields a Schwinger integral of the form

$$\int_{2\pi\alpha'}^\infty dT T^{-\frac{(p-1)}{2}} \int_0^1 dy (1 - 2y)^2 e^{-T(k^2(y-y^2) + (n-1)\alpha'^{-1})}. \tag{A.5}$$

To obtain the field theory limit, we perform the integrals above and then take the $\alpha' \rightarrow 0$ limit; we can do this using the exponential integral. For example, when $p = 3$ we have the standard field theory behaviour, with

$$\Pi_{1IR}^P = \frac{d}{3} \frac{g_{D_3}^2}{(4\pi)^2} \ln k^2 + O(1). \tag{A.6}$$

For $d = 22$ we obtain the beta function of ref. [21], but for the case $d = 0$, we find that the leading logarithm cancels, and we have the finite result

$$\Pi_{1IR}^P = \frac{16}{3} \frac{g_{D_3}^2}{(4\pi)^2} + O(k^2). \tag{A.7}$$

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